A SUMMATION FORMULA FOR APPELL'S FUNCTION F_2

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ABSTRACT

The exact solution of number of problems in quantum mechanics has been given in terms of Appell's function F_2; in an extension of this work I have given here a summation formula, which is as follows:

$$\sum_{n=0}^{m} F_2(a,-n,-n;1,1;x,y)$$

$$=\frac{(m+1)(x-y)^{-1}}{a}[F_2(a-1,-m,-m-1;1,1;x,y)-\rightleftharpoons],$$

where $\rightleftharpoons$ shows the presence of a similar term with x and y interchanged.

The exact solution of number of problems in quantum mechanics has been given in terms of Appell's function F_2; in an extension of this work, I give here, a summation formula, which may prove to be useful.

We take for F_2 the definition

(1) $F_2(a,b,b';c,c';x,y)$

$$=\frac{1}{\Gamma(a)}\int_0^{\infty} e^{-t}\, t^{a-1}\, {}_1F_1(b;c;xt)\,{}_1F_1(b';c';yt)dt,$$

$$|x|+|y|<1 \text{ and } Rl(a)>0.$$

A special case of (1) is

(2) $F_2(a,-n,-n;1,1;x,y)$

$$=\frac{1}{\Gamma(a)}\int_0^{\infty} e^{-t}t^{a-1}L_n(xt)L_n(yt)dt,$$

where $L_n(x)={}_1F_1(-n;1;x)$.

Therefore

$$\sum_{n=0}^{m} F_2(a,-n,-n;1,1;x,y)$$

$$=\frac{1}{\Gamma(a)}\sum_{n=0}^{m}\int_0^{\infty} e^{-t}t^{a-1}L_n(xt)L_n(yt)dt.$$

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The change in order of summation and integration is easily justified and we get the right-hand side as

$$=\frac{1}{\Gamma(a)}\int_0^\infty e^{-t}t^{a-1}\sum_{n=0}^{m} L_n(xt)L_n(yt)dt.$$

Using, now the relation [1, pp. 214],

(3) $$\sum_{k=0}^{n} L_k(x)L_k(y)=(n+1)(x-y)^{-1}[L_{n+1}(y)L_n(x)-L_{n+1}(x)L_n(y)],$$

the right-hand side becomes

$$=\frac{(m+1)(x-y)^{-1}}{\Gamma(a)}\int_0^\infty e^{-t}t^{a-2}[L_{m+1}(yt)L_m(xt)-L_{m+1}(xt)_{L_m(yt)}]dt, R1(a)>1.$$

Now separating the right-hand side as the difference of two integrals then by virtue of (1), we get

(4) $$\sum_{n=0}^{m} F_2(a,-n,-n;1,1;x,y)$$

$$=\frac{(m+1)(x-y)^{-1}}{a}[F_2(a-1,-m,-m-1;1,1;x,y)-\rightleftharpoons],$$

where $\rightleftharpoons$ shows the presence of a similar term with x and y interchanged.

This is a summation formula.

References

1. Earl D. Rainville, *Special functions*, (1960) New York.

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